REGULAR REGIME FOR A HOLLOW CYLINDER HEATED AT ITS INNER SURFACE

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The initial stage of the regular regime is analyzed. The features found in the evolution of this stage may be useful in research on thermal physics.

The classical approach to the study of the heating of an object in a medium with a constant temperature involves the distinction of three regimes: 1) the initial or disordered regime; 2) the regular regime; 3) the steady-state regime [1, 2].

In the regular regime, the temperature changes are described by a simple exponential function and are independent of the initial temperature distribution.

The concept of the regularization of the heating process is also extended to the cases in which the temperature field in the limiting steady state is nonuniform [3].

All definitions of the regular regime are based on solutions which are series, each term of which contains an exponential factor. Depending on the particular boundary conditions, the solution may also contain terms in which the time appears without an exponential function. In this representation, a single term (the first term), which contains the time, emerges as the most important term at a certain time after the beginning of the process. After this time, the temperature field can be described by a very simple function (exponential, linear, etc.), in accordance with the boundary conditions.

An experimental confirmation of the theory for the regular regime actually means that the sensitivity of the measuring instruments is limited and that, beginning at a certain time, the components of the temperature field lying below the sensitivity of the instruments are not detected. Furthermore, even if the instrument does detect deviations from the regular regime, they can be neglected within some permissible error. In other words, from the entire spectrum of the temperature field we distinguish that component which is the most important. The time t_{min} marking the beginning of the regular regime can be determined only within a certain error, and this time depends on this error [2].

Remaining within the framework of these arguments, we can replace the exponential function by a series in increasing powers of the time. We can evidently always find a time t_{max} such that for all $t < t_{max}$ we can retain in the expansion of the exponential function only the term containing the first power of the time, within a specified error; in other words, in the initial stage of the regular regime ($t_{min} < t < t_{max}$), the time dependence of the temperature can be treated as linear. Below we refer to this as the "quasilinear" stage.

The length of this quasilinear stage should evidently be determined from the equation $\Delta t_l = t_{max} - t_{min}$; this result depends on several factors, including the accuracy with which t_{min} and t_{max} are determined.

Let us examine certain features in the evolution of the quasilinear stage for the case of an infinite hollow cylinder ($R_0 < r < R$), whose inner surface begins to experience a heat source of constant strength Q_1 (per unit length) at time $t = 0$. The cylinder is initially at the temperature of the surrounding medium.

In this case we can write the solution of the problem, in dimensionless variables, as

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$$\Theta = \frac{1}{2\rho_0} \sum_{m=1}^{\infty} \frac{G(s_m\rho)}{s_m[(\mathrm{Bi}^2 + s_m^2)\Phi_m^2 - 1]} [1 - \exp(-s_m^2 \mathrm{Fo})], \quad (1)$$

where

$$\Theta = \frac{T - T_0}{Q_1} \lambda;$$

$$G(s_m\rho) = J_1(s_m\rho_0) Y_0(s_m\rho) - Y_1(s_m\rho_0) J_0(s_m\rho);$$

$$\Phi_m = \frac{J_1(s_m\rho_0)}{s_m J_1(s_m) - \mathrm{Bi}\, J_0(s_m)}.$$

The numbers s_m for the summation over m = 1, 2, 3, ... are defined as the positive roots of the equation

$$\mathrm{Bi}\, G(s) + sG'(s) = 0. \quad (2)$$

The roots of this equation form a discrete increasing sequence $s_1 < s_2 < s_3 < \ldots$, so that for any arbitrarily small positive number ε we can find a number Fo_{min}, such that for all $\mathrm{Fo} \geq \mathrm{Fo}_{min}$ we have the inequality

$$\exp(-s_2^2 \mathrm{Fo}) \leqslant \varepsilon, \quad (3)$$

under which we can retain simply a single term containing the time in Solution (1). In other words, we can use

$$\Theta = \Theta_\infty - D_1 \exp(-s_1^2 \mathrm{Fo}), \quad (4)$$

where

$$\Theta_\infty = \frac{1}{2\rho_0} \sum_{m=1}^{\infty} \frac{G(s_m\rho)}{s_m[(\mathrm{Bi}^2 + s_m^2)\Phi_m^2 - 1]} = \frac{1}{2\pi \mathrm{Bi}} (1 - \mathrm{Bi} \ln \rho);$$

$$D_1 = \frac{G(s_1\rho)}{2\rho_0 s_1 [(\mathrm{Bi}^2 + s_1^2)\Phi_1^2 - 1]}.$$

From Inequality (3) we find

$$\mathrm{Fo}_{min} = \frac{1}{s_2^2} \ln \frac{1}{\varepsilon}, \quad (5)$$

at which Dependence (4), describing a regular temperature increase, becomes valid, within the permissible error.

It can be shown that the root s_2 reaches its minimum value at Bi = 0 and $\rho_0 = 0$ (a solid cylinder): $s_2 = 3.8317$. In this case the maximum time before the regular regime is reached, e.g., for $\varepsilon = 10^{-2}$, is given by

$$\mathrm{Fo}_{min} \approx 0.32.$$

In the case Bi = 0, the regular regime is characterized by a linear time dependence of the temperature (adiabatic heating).

The first root in Eq. (2), s_1, is always smaller than s_2, and it vanishes at Bi = 0. Accordingly, for any arbitrarily small positive number ε and for an arbitrarily large number $\mathrm{Fo}_{max} > \mathrm{Fo}_{min}$ we can find a value s_1 such that for all $\mathrm{Fo} \leq \mathrm{Fo}_{max}$ we have the inequality

$$\frac{1}{2} s_1^2 \mathrm{Fo} \leqslant \varepsilon \quad (\varepsilon \ll 1), \quad (6)$$

under which Dependence (4) can be replaced, within a permissible error, by the linear dependence

$$\Theta = D + A\,\mathrm{Fo}, \quad (7)$$

where

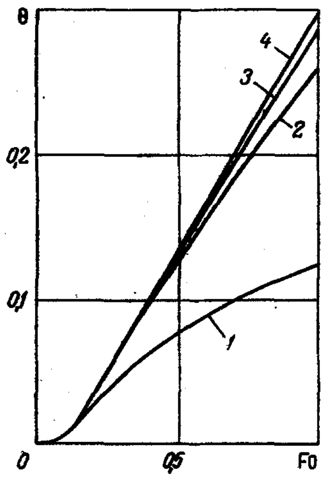

Fig. 1

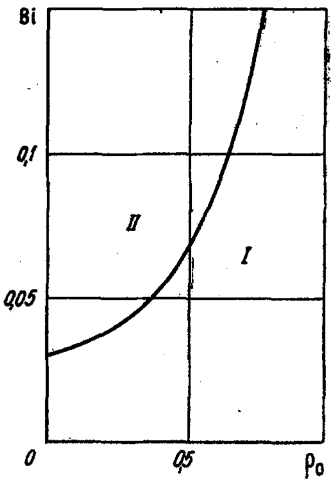

Fig. 2

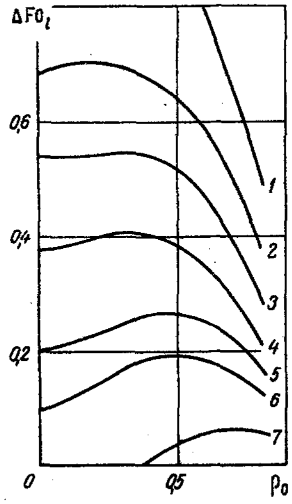

Fig. 3

Fig. 1. Dimensionless temperature as a function of the Fourier number at the outer surface of the cylinder.

Fig. 2. Regions in which the quasilinear stage does and does not exist for the case $\varepsilon = 10^{-2}$.

Fig. 3. Length of the quasilinear stage as a function of ρ_0 in the case $\varepsilon = 10^{-2}$ for various values of the Biot number Bi.

$$D = \Theta_\infty - D_1, \quad A = D_1 s_1^2.$$

From (6) we find

$$\mathrm{Fo}_{\max} = \frac{2\varepsilon}{s_1^2}\ .$$

Accordingly, in the range $\mathrm{Fo}_{\min} < \mathrm{Fo} < \mathrm{Fo}_{\max}$ the function $\Theta(\mathrm{Fo})$ can be assumed linear with the specified accuracy (this is the quasilinear stage). The slope of the quasilinear stage is governed by the quantity $A = D_1 s_1^2$, and its duration is governed by

$$\Delta \mathrm{Fo}_l = \frac{2\varepsilon}{s_1^2}\left[1 - \frac{1}{2\varepsilon}\left(\frac{s_1}{s_2}\right)^2 \ln\frac{1}{\varepsilon}\right]. \tag{8}$$

Since the roots s_1 and s_2 are functions of ρ_0 and Bi, then for a fixed quantity ε the quantity $\Delta\mathrm{Fo}_l$ is also a function of these parameters. In the limit $\mathrm{Bi} \to 0$, we have $s_1 \to 0$, while $\Delta\mathrm{Fo}_l \to \infty$; i.e., the exponential function degenerates into a purely linear function (adiabatic heating).

As an example we show in Fig. 1 the functions $\Theta(\mathrm{Fo})$ for the outer surface ($\rho = 1$) for $\rho_0 = 0.2$ and Bi = 1.0 (1), Bi = 0.1 (2), Bi = 0.01 (3), and Bi = 0 (4).

Since the parameters ρ_0 and Bi are independent, there is a possible combination of these parameters for which we would have $\Delta\mathrm{Fo}_l = 0$. Figure 2 shows a curve relating Bi and ρ_0, under the condition $\Delta\mathrm{Fo}_l = 0$ ($\varepsilon = 10^{-2}$). For any combination of the parameters Bi and ρ_0 in region I (below the curve), a quasilinear stage exists (for the value of ε adopted). In region II (above the curve), there is no quasilinear stage. It follows from Fig. 2 that as ρ_0 is increased the range of values of Bi for which a quasilinear stage exists becomes broader.

Figure 3 shows the length of the quasilinear stage as a function of ρ_0 for Bi = 0.0075 (1), 0.01 (2), 0.012 (3), 0.015 (4), 0.02 (5), 0.025 (6), and 0.05 (7) for $\varepsilon = 10^{-2}$. We see that a maximum appears on the $\Delta\mathrm{Fo}_l(\rho_0)$ dependence as Bi is increased. In the limit $\rho_0 \to 1$, all the calculated curves tend toward zero; this happens because (as can be shown) in the limit $\rho_0 \to 1$ the roots s_1 and s_2 tend toward infinity (regardless of $\mathrm{Bi} \neq 0$), but the inequality $s_1 < s_2$ remains valid.

The quasilinear heating regime described by (7) is analogous to the quasisteady regime (see, e.g., [4, 5]). The quasisteady regime, however, occurs upon a linear change in the temperature of the medium or the surface, while the quasilinear stage occurs at a constant temperature of the medium. In the quasilinear regime the slope of the function $\Theta(Fo)$ is governed by the quantity $D_1 s_1^2$, itself a function of the Biot number, and the slope is different at different points over the cross section of the cylinder. In the quasisteady regime, on the other hand, this slope is the same everywhere and is governed by the rate of change of the temperature of the medium or the surface. Furthermore, the linear stage in the quasilinear regime is of only limited duration.

These arguments can be extended to other objects of simple shape (a plane or a sphere, for example) under analogous conditions.

The theoretical conclusions reached in this study have been tested experimentally, and an exceptionally simple procedure has been worked out for determining several thermal properties.

NOTATION

Θ, dimensionless temperature; $\rho = r/R$, dimensionless radial coordinate; $\rho_0 = R_0/R$, dimensionless inner radius; $Bi = (\alpha/\lambda)R$, Biot number; $Fo = (a/R^2)t$, Fourier number; $J_0(s\rho)$, $J_1(s\rho)$, $Y_0(s\rho)$, $Y_1(s\rho)$, Bessel functions of the first and second kind.

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